**Logical Reasoning and Counterexamples**

**Conditional Statements** A conditional statement is a statement of the form *If A, then B*. Statements in this form are called if-then statements. The part of the statement immediately following the word *if* is called the hypothesis. The part of the statement immediately following the word *then* is called the conclusion.

**Example 1** Identify the hypothesis and conclusion of each statement.

a. If it is Wednesday, then Jerri has aerobics class.
   - Hypothesis: it is Wednesday
   - Conclusion: Jerri has aerobics class

b. If $2x - 4 < 10$, then $x < 7$.
   - Hypothesis: $2x - 4 < 10$
   - Conclusion: $x < 7$

**Example 2** Identify the hypothesis and conclusion of each statement. Then write the statement in if-then form.

a. You and Marylynn can watch a movie on Thursday.
   - Hypothesis: it is Thursday
   - Conclusion: you and Marylynn can watch a movie
   - If it is Thursday, then you and Marylynn can watch a movie.

b. For a number $a$ such that $3a + 2 = 11$, $a = 3$.
   - Hypothesis: $3a + 2 = 11$
   - Conclusion: $a = 3$
   - If $3a + 2 = 11$, then $a = 3$.

**Exercises**

Identify the hypothesis and conclusion of each statement.

1. If it is April, then it might rain.
2. If you are a sprinter, then you can run fast.
3. If $12 - 4x = 4$, then $x = 2$.
4. If it is Monday, then you are in school.
5. If the area of a square is 49, then the square has side length 7.

Identify the hypothesis and conclusion of each statement. Then write the statement in if-then form.

6. A quadrilateral with equal sides is a rhombus.

7. A number that is divisible by 8 is also divisible by 4.

8. Karlyn goes to the movies when she does not have homework.
Logical Reasoning and Counterexamples

Deductive Reasoning and Counterexamples  Deductive reasoning is the process of using facts, rules, definitions, or properties to reach a valid conclusion. To show that a conditional statement is false, use a counterexample, one example for which the conditional statement is false. You need to find only one counterexample for the statement to be false.

Example 1 Determine a valid conclusion from the statement *If two numbers are even, then their sum is even* for the given conditions. If a valid conclusion does not follow, write *no valid conclusion* and explain why.

a. The two numbers are 4 and 8.
   4 and 8 are even, and $4 + 8 = 12$. Conclusion: The sum of 4 and 8 is even.

b. The sum of two numbers is 20.
   Consider 13 and 7. $13 + 7 = 20$
   However, $12 + 8$, $19 + 1$, and $18 + 2$ all equal 20. There is no way to determine the two numbers. Therefore there is no valid conclusion.

Example 2 Provide a counterexample to this conditional statement. *If you use a calculator for a math problem, then you will get the answer correct.*
Counterexample: If the problem is $475 \div 5$ and you press $475 - 5$, you will not get the correct answer.

Exercises

Determine a valid conclusion that follows from the statement *If the last digit of a number is 0 or 5, then the number is divisible by 5* for the given conditions. If a valid conclusion does not follow, write *no valid conclusion* and explain why.

1. The number is 120.

2. The number is a multiple of 4.

3. The number is 101.

Find a counterexample for each conditional statement.

4. If Susan is in school, then she is in math class.

5. If a number is a square, then it is divisible by 2.

6. If a quadrilateral has 4 right angles, then the quadrilateral is a square.

7. If you were born in New York, then you live in New York.

8. If three times a number is greater than 15, then the number must be greater than six.

9. If $3x - 2 \leq 10$, then $x < 4$. 
Logical Reasoning and Counterexamples

Identify the hypothesis and conclusion of each statement.

1. If it is raining, then the meteorologist’s prediction was accurate.

2. If \( x = 4 \), then \( 2x + 3 = 11 \).

Identify the hypothesis and conclusion of each statement. Then write the statement in if-then form.

3. When Joseph has a fever, he stays home from school.

4. Two congruent triangles are similar.

Determine whether a valid conclusion follows from the statement: If two numbers are even, then their product is even for the given condition. If a valid conclusion does not follow, write no valid conclusion and explain why.

5. The product of two numbers is 12.

6. Two numbers are 8 and 6.

Find a counterexample for each conditional statement.

7. If the refrigerator stopped running, then there was a power outage.

8. If \( 6h - 7 < 5 \), then \( h \leq 2 \).

9. GEOMETRY Consider the statement: If the perimeter of a rectangle is 14 inches, then its area is 10 square inches.
   a. State a condition in which the hypothesis and conclusion are valid.
   b. Provide a counterexample to show the statement is false.

10. ADVERTISING A recent television commercial for a car dealership stated that “no reasonable offer will be refused.” Identify the hypothesis and conclusion of the statement. Then write the statement in if-then form.
1. KINDERGARTEN Identify the hypothesis and conclusion and write the statement in if-then form.

Helene will go to school when she is five years old.

2. GEOMETRY Write a valid conclusion that follows from the statement below for the given condition. If a valid conclusion does not follow, write no valid conclusion and explain why.

If the radius of a circle is multiplied by 10, its area is multiplied by 100.

Circle A has a radius of 5 centimeters and an area equal to 78.5 square centimeters, while circle B has a radius of 50 centimeters.

3. PRIME NUMBERS For centuries, mathematicians have tried to develop a formula to generate prime numbers. Legendre and Euler each came up with a number of polynomial formulas that generate primes. Consider the following conditional statement and find a counterexample to show that it is not always true.

If $n$ is a whole number, $2n^2 + 11$ is a prime number.

4. QUADRILATERALS The Venn diagram shows the relationships of various quadrilaterals.

State whether each statement is valid. If it is not valid, write a new statement that is valid.

a. If a square is a rhombus and a square is a rectangle, then a rhombus is a rectangle.

b. If a quadrilateral is not a parallelogram, it is a trapezoid.

c. If a quadrilateral is not a square, it is not a rhombus.